

# Mapping the proton drip line in the suburanium region and for superheavy elements

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The relativistic Hartree-Bogoliubov (RHB) model is employed in the mapping of the proton drip line in the suburanium region and for superheavy elements. The RHB prediction for the last bound isotope of each odd- $Z$  element in these mass regions, and the ground-state quadrupole deformations of nuclei at the drip line, is compared with several mass formulas and with recent experimental data on the location of the proton drip line in the suburanium region.

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Relativistic mean-field models have been very successfully employed in analyses of a variety of nuclear structure phenomena, not only in nuclei along the valley of  $\beta$  stability, but also in exotic nuclei with extreme isospin values and close to the particle drip lines. The relativistic Hartree-Bogoliubov (RHB) model, based on the relativistic mean-field theory and on the Hartree-Fock-Bogoliubov framework, provides a unified description of mean field and pairing correlations, which is particularly important for the structure of very weakly bound nuclei at the particle drip lines. In proton-rich nuclei the RHB model has been used to map the drip line from  $Z=31$  to  $Z=73$ , and to investigate the phenomenon of ground-state proton radioactivity [1–4]. The location of the proton drip line, the ground-state quadrupole deformations and one-proton separation energies at and beyond the drip line, the deformed single-particle orbitals occupied by the odd valence proton, and the corresponding spectroscopic factors have been compared with available experimental data.

The phenomenon of proton emission from ground states of nuclei in the region  $53 \leq Z \leq 73$  has been the subject of numerous experiments and theoretical analyses in the last decade. An important issue in future experimental studies of heavy proton-rich nuclei is the possible observation of ground-state proton emission in the suburanium region. It would be clearly desirable to have a detailed experimental knowledge and theoretical understanding of the evolution of the proton drip line for heavy proton-rich nuclei. In Fig. 1 we display the results of the relativistic Hartree-Bogoliubov calculation for the proton drip line of odd- $Z$  nuclei with  $73 \leq Z \leq 91$ . The NL3 effective interaction [5] is used for the mean-field Lagrangian, and pairing correlations are described by the pairing part of the finite range Gogny interaction D1S [6]. This particular combination of effective forces in the  $ph$  and  $pp$  channels has been used in most of our recent applications of the RHB theory. A detailed description of our RHB model for axially deformed nuclei can be found, for instance, in Ref. [2]. The RHB equations are solved self-consistently, with potentials determined in the mean-field ap-

proximation from solutions of Klein-Gordon equations for the meson fields. The Dirac-Hartree-Bogoliubov equations and the equations for the meson fields are solved by expanding the nucleon spinors and the meson fields in terms of the eigenfunctions of a deformed axially symmetric oscillator potential. A simple blocking procedure is used in the calculation of odd-proton and/or odd-neutron systems. The blocking calculations are performed without breaking the time-reversal symmetry. The RHB (NL3+D1S) calculation predicts the last bound isotopes for each element. Nuclei to the left are proton unstable. This is a unique procedure for the determination of the proton drip line, as we have investigated the question of stability for each nucleus in the  $(N, Z)$  plane. In particular the drip line calculated in this way does not depend on the direction (constant  $N$  or  $Z$ ), in which we approach this line. For odd- $Z$  nuclei the calculated position of the proton drip line can be compared with very recent experimental data [7]. An excellent agreement between theory and

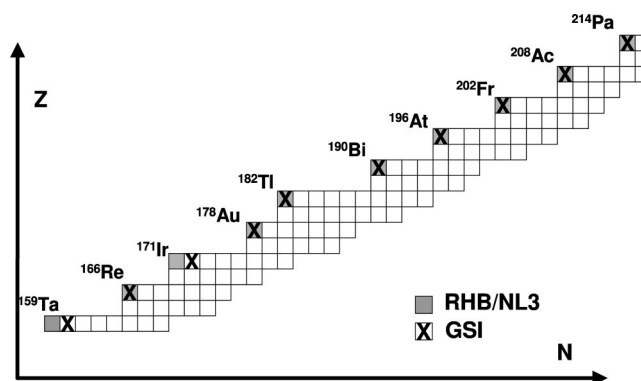


FIG. 1. The proton drip line in the suburanium region  $73 \leq Z \leq 91$ . The squares denote the calculated position of the last bound isotope for each odd- $Z$  element. Nuclei to the left are predicted to be proton unstable by the present RHB (NL3+D1S) calculation. The experimentally determined location of the proton drip line [7] is denoted by crosses.

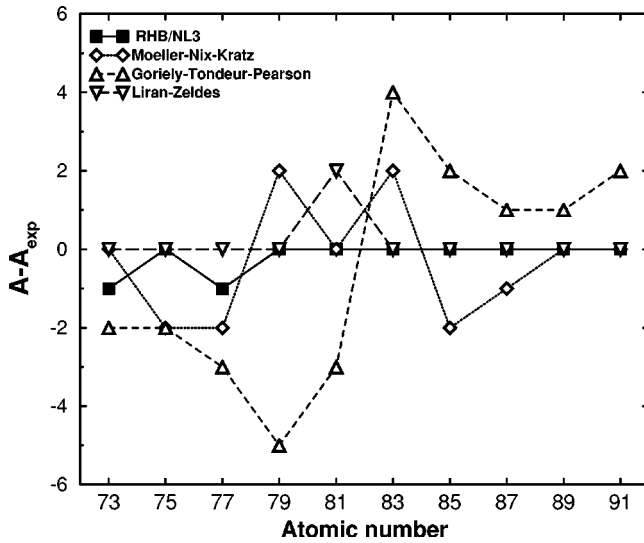


FIG. 2. The proton drip line in the suburanium region  $73 \leq Z \leq 91$ . The mass-number differences between the predicted last bound isotope for each odd- $Z$  element, and the experimentally determined location of the proton drip line, are plotted for the present RHB (NL3+D1S) calculation, in comparison with the mass models of Refs. [9–11].

experiment is found. Only for Ta and Ir the RHB model prediction differs by one mass unit from the experimentally determined position of the proton drip line.

In Fig. 2 we compare the fully self-consistent microscopic RHB (NL3+D1S) prediction for the location of the proton drip line with those of the mass models: the finite-range droplet mass (FRDM) model [9] of Möller, Nix, and Kratz; the Hartree-Fock nuclear mass table of Goriely, Tondeur, and Pearson [10]; and the semiempirical shell-model mass equation of Liran and Zeldes [11]. The mass-number differences between the predicted last bound isotope for each odd- $Z$  element, and the experimentally determined location of the proton drip line are plotted as functions of the atomic number. The predictions of the RHB model and the Liran-Zeldes mass formula are in very good agreement with the experimental proton drip line in this mass region. As is also shown in Fig. 1, only for Ta and Ir the RHB (NL3+D1S) drip line differs by one mass unit from the experimental drip line. The prediction of the Liran-Zeldes mass formula differs only for Tl by two mass units from the experimentally determined position of the proton drip line. The other two mass models do not compare so well with the experimental data. Except for Tl, in all cases the drip line prediction of the FRDM differs by one or two units from the experimental drip line, whereas the drip line predicted by the Hartree-Fock nuclear mass table [10] displays much larger discrepancies, especially for Au (−5 mass units) and for Bi (4 mass units). For the nuclei at the experimentally determined drip line [7], in Table I we compare the one-proton separation energies predicted by the present RHB (NL3+D1S) calculation, the FRDM model [9], and the Hartree-Fock nuclear mass table [10].

In the two panels on the left of Fig. 3 we display the calculated ground-state quadrupole deformations of nuclei at

TABLE I. Proton separation energies  $S_p$  predicted by the RHB (NL3+D1S) calculation, the FRDM model [9], and the Hartree-Fock nuclear mass table (HF NMT) [10], for the nuclei at the observed proton drip line [7] in the region  $73 \leq Z \leq 91$

Drip line nucleus	$S_p$ (MeV)		
	RHB	FRDM	HF NMT
$^{160}\text{Ta}$	0.49	0.54	0.50
$^{166}\text{Re}$	0.56	0.64	0.60
$^{172}\text{Ir}$	0.38	0.33	0.70
$^{178}\text{Au}$	0.43	−0.01	0.80
$^{182}\text{Tl}$	0.20	0.25	1.10
$^{190}\text{Bi}$	0.13	−0.35	−0.60
$^{196}\text{At}$	0.17	0.40	−0.30
$^{202}\text{Fr}$	0.38	0.45	−0.20
$^{208}\text{Ac}$	0.29	0.05	−0.10
$^{214}\text{Pa}$	0.15	0.20	0.20

the drip lines in the sub-uranium region  $73 \leq Z \leq 91$ . The RHB (NL3+D1S) self-consistent ground-state  $\beta_2$  values are plotted in the upper left panel as function of the atomic number, and compared with the quadrupole deformations predicted by the finite-range droplet model [9] and the Hartree-Fock-Skyrme calculations of Goriely *et al.* [10] (lower left panel). All three models predict similar deformations: the proton drip line in this mass region is characterized by small ground-state deformations and shape transitions. We note, in particular, the prolate-oblate shape transition between Ir and Au: all drip line nuclei with  $Z > 77$  are slightly oblate deformed.

Model predictions of the location of the proton drip line are also very important for experimental studies of the synthesis and stability of the heaviest elements. All the recently

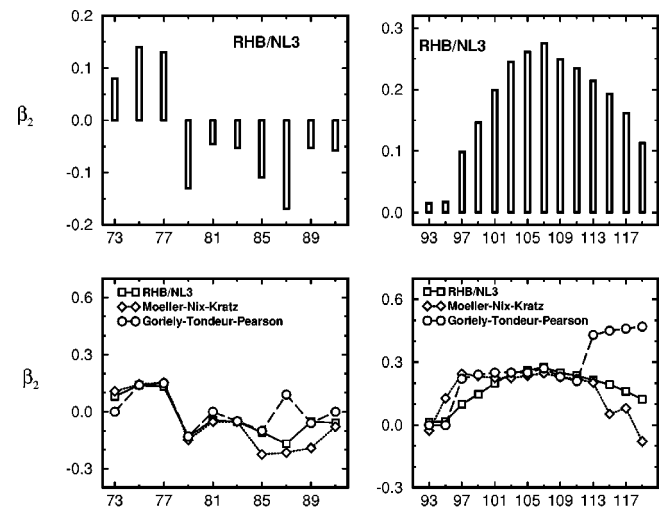


FIG. 3. Self-consistent RHB (NL3+D1S) ground-state quadrupole deformations for nuclei at the proton drip line in the suburanium region  $73 \leq Z \leq 91$  (upper left panel) and for superheavy nuclei (upper right panel). The comparison with the predictions of the models from Refs. [9,10] is shown in the lower panels.

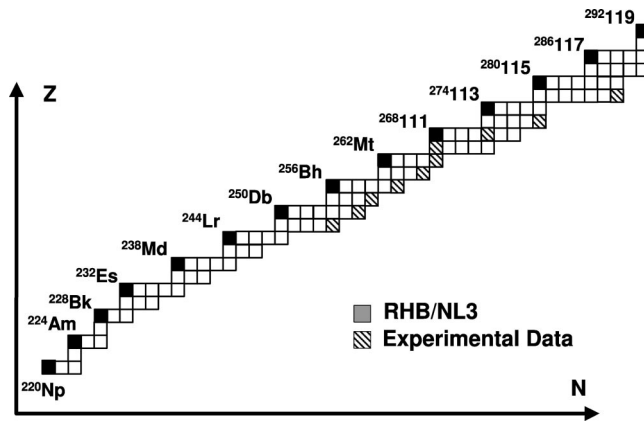


FIG. 4. The proton drip line in the region of superheavy elements  $93 \leq Z \leq 119$ . The black squares denote the calculated location of the last bound isotope for each odd- $Z$  element. The superheavy nuclei ( $Z > 103$ ) which have been synthesized in experiment [8] are denoted by white crossed squares.

discovered isotopes of the superheavy elements, in particular, belong to very proton rich systems. In Fig. 4 we display the section of the chart of the nuclides along the proton drip line in the region  $93 \leq Z \leq 119$ . The RHB (NL3+D1S) prediction for the last bound isotope of each odd- $Z$  element is shown together with the experimentally known superheavy nuclei ( $Z > 103$ ) [8]. The corresponding ground-state quadrupole deformations of the drip line nuclei are shown in the panels on the right of Fig. 3: the RHB (NL3+D1S) results in the upper right panel, and the comparison with the FRDM [9] and Hartree-Fock-Skyrme [10]  $\beta_2$  values (lower right panel).

The drip line nuclei in this mass region are characterized by pronounced quadrupole deformations, especially for  $101 \leq Z \leq 113$ . For heavier drip line nuclei the RHB and FRDM predict a gradual decrease of deformation, possibly indicating the occurrence of a spherical region around  $Z=120$ . The Hartree-Fock-Skyrme plus BCS calculation of Ref. [10], on the other hand, predicts an unexpected increase of deformation for  $Z \geq 113$  to large positive values of  $\beta_2$  that are characteristic for superdeformed shapes.

In conclusion, we have applied the RHB in the analysis of the proton drip line in the suburanium region  $73 \leq Z \leq 91$  and for superheavy elements  $93 \leq Z \leq 119$ . The NL3 effective interaction has been used in the mean-field Lagrangian, and pairing correlations have been described by the pairing part of the finite range Gogny interaction D1S. The RHB (NL3+D1S) prediction for the last bound isotope of each odd- $Z$  element in these mass regions, and the ground-state quadrupole deformations of nuclei at the drip line, has been compared with several mass formulas and with recent experimental data on the location of the proton drip line in the suburanium region. An excellent agreement has been found between the drip line calculated with the RHB model and the experimentally determined drip line in the region  $73 \leq Z \leq 91$ . The RHB model also predicts that the experimentally known superheavy nuclei are located close to the proton drip line.

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- [1] D. Vretenar, G. A. Lalazissis, and P. Ring, Phys. Rev. Lett. **82**, 4595 (1997).
- [2] G. A. Lalazissis, D. Vretenar, and P. Ring, Nucl. Phys. **A650**, 133 (1999).
- [3] G. A. Lalazissis, D. Vretenar, and P. Ring, Phys. Rev. C **60**, 051302 (1999).
- [4] G. A. Lalazissis, D. Vretenar, and P. Ring, Nucl. Phys. **A679**, 481 (2001).
- [5] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C **55**, 540 (1997).
- [6] J. F. Berger, M. Girod, and D. Gogny, Nucl. Phys. **A428**, 32

(1984).

- [7] Yu. Novikov *et al.*, Nucl. Phys. **A697**, 92 (2002).
- [8] S. Hofmann and G. Münzenberg, Rev. Mod. Phys. **72**, 733 (2000).
- [9] P. Möller, J. R. Nix, and K.-L. Kratz, At. Data Nucl. Data Tables **66**, 131 (1997).
- [10] S. Goriely, F. Tondeur, and J. M. Pearson, At. Data Nucl. Data Tables **77**, 311 (2001).
- [11] S. Liran and N. Zeldes, At. Data Nucl. Data Tables **17**, 431 (1976).